TOPICS COVERED for Probability and Statistics Theory (PhD preliminary examination)

July 27, 2016

General Background (A) (1) combinatorial analysis and axioms of probability (2) elementary random variable theory: expectation, variance, moments, distribution function, probability density functions etc (3) major discrete random variables: Bernoulli, binomial, Poisson, geometric major continuous random variables: uniform, normal, exponential (4) Applications and simple statistical modelling (5) conditional probability: Bayes formula, independence, conditional expectation, prediction, iid random variables (6) joint distributions, joint distribution functions, marginal distributions, (7) Moment generating functions and applications.

Measure theory (B) (1) elementary measure theory, σ - algebras, probability triples, continuity of probabilities, integration, L^p spaces, monotone convergence theorem, Lebesgue measure, Borel-Cantelli lemmas, Chebychevs inequality, Markovs inequality, weak law of large numbers, strong law of large numbers, (2) Modes of Convergence- almost sure convergence, convergence in probability, L^p convergence, convergence in distribution.

Markov chains and random walks (C) Markov chain theory (1) first step analysis (2) transience, recurrence and irreducibility (3) stationary distributions and existence theorems (4) random walks as Markov chains.

Statistics Theory, Estimation and Hypothesis Testing (D) Statistics theory (1) Bias and unbiased estimation, (2) confidence interval, (3) likelihood, sufficient statistics, maximum likelihood estimate, (4) generalized linear model, exponential family distributions, Fishers information matrix, (5) large sample theory and consistency, (7) hypothesis testing, type I and type II errors, likelihood ratio test.

Recommended Texts:
- A First Look at Rigorous Probability Theory by Jeffrey Rosenthal, 2000. (B,C).

- An Introduction to Stochastic Modelling, Karlin and Taylor, 3rd Edition, 1998, Academic Press.
- A First Course in Probability, Sixth Edition by Sheldon Ross, 2002, Prentice Hall (A).
- Probability: theory and Examples, 3rd Edition, Richard Durrett, Duxbury Press.
- An Introduction to Probability Theory and Its Applications, Vol 1, 3rd edition, 1968 by William Feller (any edition would be fine) (A,C).
- Probability by Leo Breiman, 1968, Addison- Wesley (B).
- Mathematical Statistics and Data Analysis, John Rice 2nd (Duxbury) or 3rd (Cengage) edition.
- Mathematical Statistics, Basic Ideas and Selected Topics, Vol 1, 2nd edition Peter Bickel and Kjell Doksum, Pearson.
- Generalized Linear Models, 2nd edition, Peter McCullagh and John Nelder. Chapman /CRC.